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UNIVERSAL REGULARITIES OF HYDRAULIC FRACTURING WITHIN THE FRAMEWORK OF THE PERKINS-KERN-NORDGREN MODEL

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An algorithm that permits one to find a solution of the Perkins-Kern-Nordgren problem for standard regimes of hydraulic fracturing is suggested. The universal qualitative specifics of the behavior of a crack is shown; in particular, the asymptotics of crack opening at the tip is found, and it is shown that the character of the crack is determined by the rate of increase or decrease in the crack size.

The design of hydraulic fracturing of a productive reservoir is based on results of a theoretical analysis of three basic models [1]. They differ by the geometry of a developing crack: a rectilinear crack in a plane, a circular crack in space, and a ribbon-shaped crack growing in the horizontal direction of the ribbon whose plane is oriented vertically. The last model, which is usually called a Perkins-Kern-Nordgren (PKN) model, is developed to a greater extent. In the simplest case of the propagation of a crack of a constant height where a viscous fluid is injected into it, the Nordgren equation [2] holds. The fracture fluid is injected from a source in the center. Within the framework of the given model, the strength properties of the medium are ignored.

In previous studies of the PKN model, due attention has not been given to the fact that the tip of a crack is the singular point of the Nordgren equation. Generally speaking, a boundary layer is formed in the neighborhood of the singular point in which the solution changes rapidly, and, consequently, standard finite-difference methods work poorly. To present difficulties that encounter, we shall turn, for example, to one of the boundary conditions for the Nordgren equation, namely, the equality to zero of the gradient of the fourth degree of opening (pressure) at the boundary. It corresponds to the impermeability through the crack tip. In the simplest difference approximation of the derivative at the boundary, the pressure at a distance of the grid step from the tip coincides automatically with the pressure at the boundary, whereas, in reality it should vary abruptly.

An attempt to avoid this difficulty by the formulation of a no-flow condition in the form of an integral law of conservation of mass [1] rather than in the differential form only disguises it if special measures are not taken, but does not eliminate it.

If the asymptotic behavior of the pressure in a boundary layer, which is closely connected with the law of crack growth, is not taken into account in a numerical calculation, this can have a significant effect on the accuracy of determination of the crack velocity. We shall illustrate this statement on the problem of a large crack with a uniform grid, when the longitudinal size of the boundary layer can be considerably smaller than the grid step. It is possible to make an attempt to approximate its abrupt change in a thin boundary layer by a pressure jump at the boundary. However, such a scheme is incorrect, because an uncertain parameter (the magnitude of the jump) appears and the variation of this parameter changes the velocity. There exists a magnitude of the jump at which the crack stops, the law of conservation of mass being fulfilled.

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It is important that the errors in the velocity, accumulated during the prehistory of hydraulic fracturing, have an effect on the basic design parameter, namely, the crack size. Finding an explicit form of the asymptotics at the tip eliminates difficulties in the determination of the crack velocity.

1. We shall consider a symmetric crack of height $2H$ and length $2L$. The crack develops in a permeable elastic medium. The X axis is assumed to be the direction of crack growth in a horizontal plane. The coordinate origin is located in the center of the crack. The fluid pressure is assumed to be constant in height and to vary slowly along the X axis, i.e., the profile of crack opening corresponds to a constant pressure, and it is elliptic in each vertical cross section (X = const). The maximum opening $2W(X,L)$ in this cross section is on the horizontal axis of the ellipse, and it is proportional to the exceeding of the pressure $P(X,L) - P_c$ over the pressure of crack opening (closure) P_c [1]:

$$P - P_c = \frac{DW}{H}, \qquad D = \frac{1}{2}\frac{E}{1-\nu^2}, \tag{1.1}$$

where the proportionality coefficient D represents a combination of standard elastic constants of the medium.

In terms of the opening $W(X,L)$ and the local flow rate $Q(X,L)$ through the crack's cross section passing through the point X, the equation of a laminar flow of a fluid with viscosity μ in a thin layer, which fills in the crack hollow, can be written, with allowance for (1.1), in the form [2]

$$\frac{\partial W^4}{\partial X} = -16\mu(\pi D)^{-1}Q. \tag{1.2}$$

Let us assume [1] that the fluid losses in a permeable medium from a unit surface of a crack obey the Carter law [3], and we shall denote the fluid-loss coefficient by γ [3], and the every-second volume of fluid losses from a part of the surface located to the right of the separated cross section X by Q_γ. In accordance with the Carter law, the fluid losses Q_γ can be presented in the form

$$Q_\gamma = 4\gamma H \int\limits_X^L \frac{dX'}{\sqrt{T-T'}},$$

where T' is the moment at which the distance from the center to the crack tip reaches the value X' and T is the current moment of time when the distance becomes equal to L. With allowance for the no-flow condition at the edge, the flow balance through the crack's hollow between the cross sections X and L can be presented in the form

$$Q = Q_\gamma + \pi H \frac{d}{dT}\int\limits_X^L W\,dX'. \tag{1.3}$$

Substituting (1.3) into (1.2) and differentiating with respect to X, we obtain the Nordgren equation [2]

$$\frac{\partial^2 W^4}{\partial X^2} = 16\mu H(\pi D)^{-1}\left(\pi\frac{\partial W}{\partial T} + \frac{4\gamma}{\sqrt{T-T'}}\right). \tag{1.4}$$

We consider that the crack length is equal to zero at the initial moment: $W(X,T)\big|_{T=0} = 0$.

Along with the initial condition, Eq. (1.4) is supplemented by three boundary conditions (one condition more than the differential order of the equation requires), which allows one to determine additionally the crack velocity.

The condition in the center of the crack is specified by the source flow rate $2Q_0$. According to (1.2), with allowance for the symmetry of the crack branches this condition is written as

$$\frac{\pi D}{16\mu}\frac{\partial W^4}{\partial X}\bigg|_{X=0} = -Q_0.$$

Two conditions are imposed at the crack edge. In essence, the condition $W(X,L)\big|_{X=L} = 0$ is the definition of the edge of a crack.

As follows from relation (1.2), the differential boundary condition, which reflects the absence of a flow through a mobile boundary, has the form

$$\left.\frac{\partial W^4(X,L)}{\partial X}\right|_{X=L} = 0.$$

In practice, three regimes of hydraulic fracturing are used most frequently. Injection is usually performed with a constant flow rate $Q_0 = Q_{01}$ to facilitate the control of hydraulic fracturing, because the interpretation of the data on the pressure P versus time T in the center of a crack becomes simpler. Then, for deriving an additional information on the crack, the pressure is frequently measured after the injection is stopped. A productive well either is closed, i.e., the fluid flow from the source in the center $Q_0 = Q_{02} = 0$ is cut off, or it works in a flowback regime with flow rate $Q_0 = Q_{03} = -\alpha Q_{01}$ ($\alpha = \text{const}$, $\alpha \sim 1$) comparable with the flow rate during injection [1].

After cessation of injection, we distinguish two stages of successive development of the crack: a stage of crack growth, when the crack length increases, and a relaxation stage which corresponds to a decrease in its length.

In determining the number of real parameters of the problem, we shall pass to the dimensionless variables $q = Q/Q_*$, $w = W/W_*$, $l = L/L_*$, $x = X/L_*$, $t = T/T_*$, and $t' = T'/T_*$, where the scale factors are of the form

$$Q_* = Q_{01}, \quad W_* = \left[\frac{\mu}{DH}\left(\frac{Q_*}{\gamma}\right)^2\right]^{1/3}, \quad L_* = \frac{\pi D}{16\mu}\frac{W_*^4}{Q_*}, \quad T_* = \left(\frac{\pi W_*}{4\gamma}\right)^2. \tag{1.5}$$

In the dimensionless variables, the equation and the initial and boundary conditions take the form

$$\frac{\partial^2 w^4}{\partial x^2} - \frac{\partial w}{\partial t} = f, \qquad f = \frac{1}{\sqrt{t-t'}}; \tag{1.6}$$

$$w\Big|_{t=0} = 0, \qquad w\Big|_{x=l} = 0; \tag{1.7}$$

$$q(x,l)\Big|_{x=l} = -\frac{\partial w^4}{\partial x}\Big|_{x=l} = 0, \qquad q(x,l)\Big|_{x=0} = -\frac{\partial w^4}{\partial x}\Big|_{x=0} = q_0. \tag{1.8}$$

According to (1.3), we present the flow rate $q(x)$ in an arbitrary cross section x in the form

$$q = \int_x^l \left(\frac{\partial w}{\partial t} + \frac{1}{\sqrt{t-t'}}\right) dx';$$

note that, in accordance with (1.2), at inner points we have

$$q(x,t) = -\frac{\partial w^4(x,t)}{\partial x}. \tag{1.9}$$

In the chosen variables, the magnitude of the flow rate q_0 in the center is equal to unity during injection, zero in the closed well, and $-\alpha$ during flowback.

It is seen from the formulation of problem (1.6)–(1.8) that, in an injection regime, the function $w(x,t)$ does not depend on the physical constants and the regime parameters. In this sense, the solution has a universal character. After the well is closed, the quantity $w(x,t,t_0)$ depends on the injection time t_0. The flowback regime is characterized by one more additional parameter α.

In the chosen variables, the Nordgren equation does not contain explicitly a parameter that would characterize the relative effect of two major factors forming the geometry of a crack: flow losses to a permeable medium and a fluid flow inside the crack. The crack length $2l$ plays this part. The fluid losses are insignificant for small l. Their effect becomes noticeable when l reaches values close to unity. The total fluid losses thus become comparable with its volume in the crack. For large l, the crack behavior is determined by fluid losses. As is known [1], in this case $w \sim l^{1/4}$.

2. We shall consider an injection regime. We shall rewrite the expression for the higher-order derivative

in Eq. (1.6) in the form

$$\frac{\partial^2 w^4}{\partial x^2} \equiv \frac{\partial}{\partial x}(4w^3)\frac{\partial w}{\partial x}.$$

Clearly, the process of crack opening is described by the nonlinear diffusion equation with power diffusivity $4w^3$ and the fluid losses-induced right-hand side. The variance of the diffusivity along the crack reflects very important properties of a hydraulic fracture. For constant diffusivity, at all moments of time, except for the initial one, the solution is formally different from zero everywhere, i.e., the crack length is infinite, and it makes no sense to speak about the rate of its increase. For variable diffusivity, its gradient at the edge is proportional to the crack velocity.

We shall consider the crack profile in the crack-tip vicinity. It is difficult to analyze Eq. (1.6) in the neighborhood of a singular point in the variables x and t, because the derivatives of w with respect to both variables tend to infinity for $x \to l$. It is better to pass to a mobile coordinate system $\xi = l - x$ with a center on the crack tip and to replace the variable t by l. From the physical viewpoint, it is natural to expect that $w(\xi, l)$ will be a smooth function of l for all values of ξ (in what follows, we shall use the former designation for the function of new variables).

We shall denote the crack velocity by $c(l) = dl/dt$. After simple manipulations we write Eq. (1.6) and the boundary conditions in the new coordinates as follows:

$$\frac{\partial^2 w^4}{\partial \xi^2} - c(l)\left(\frac{\partial w}{\partial \xi} + \frac{\partial w}{\partial l}\right) = f, \qquad f = \left[\int\limits_0^{\xi} \frac{d\xi}{c(l-\xi)}\right]^{-1/2}; \tag{2.1}$$

$$w(\xi, l)\Big|_{l=0} = 0, \quad w(\xi, l)\Big|_{\xi=0} = 0, \quad \frac{\partial w^4(\xi, l)}{\partial \xi}\Big|_{\xi=0} = 0, \quad \frac{\partial w^4(\xi, l)}{\partial \xi}\Big|_{\xi=l} = 1. \tag{2.2}$$

In this formulation, the opening, which corresponds to the current length l, depends explicitly on the crack velocity during the entire prehistory. The velocity $c(l)$ should be compatible with the formulated boundary conditions.

We note that $w \to 0$ for $\xi \to 0$ by virtue of the boundary condition, but the derivative $\partial w/\partial \xi$ increases infinitely. At the same time, $\partial w/\partial l \to 0$, which reflects the smoothness of the opening relative to the variable l. Thus, for small ξ one can expect the fullfilment of the inequality

$$\frac{\partial w(\xi, l)}{\partial l} \ll \frac{\partial w(\xi, l)}{\partial \xi}.$$

Ignoring the derivative $\partial w/\partial l$ in comparison with $\partial w/\partial \xi$ and substituting approximately $c(l-\xi)$ for $c(l)$ for small ξ, we shall reduce the equation to the one-dimensional equation $(w^4)'' - cw' - \sqrt{c/\xi} = 0$ (the differentiation with respect to ξ is primed). Having integrated this equation, with allowance for the boundary conditions $\xi = 0$ we derive the first-order equation $(w^4)' = cw + 2\sqrt{c\xi}$. For $\xi \to 0$, the function w tends to zero slower than $\sqrt{\xi}$. Therefore, leaving only the principal term in the right-hand side and cancelling out one degree of w, we rewrite the equation in the form

$$(4w^3)' = 3c. \tag{2.3}$$

It follows that the diffusivity gradient at the boundary is equal numerically to the tripled dimensionless fracture velocity.

The solution of Eq. (2.3), which vanishes at the boundary, has the form $w^3 = (3/4)c\xi$. This formula determines the asymptotic behavior of the opening at the crack tip in an injection regime, and it is seen that, in fact, the derivative $\partial w/\partial \xi$ increases unboundedly and $\partial w^4/\partial \xi \to 0$ for $\xi \to 0$.

It is noteworthy that the point is precisely the crack growth. For a negative c, there is no solution of the type considered that would have a physical meaning.

In the domain of smooth behavior of the desired functions (outside the boundary layer), in considering the injection regime, it is appropriate to solve numerically problem (2.1) and (2.2) by a standard sweep method with iterations over the nonlinearity. Here the established asymptotics allows one to determine the value of

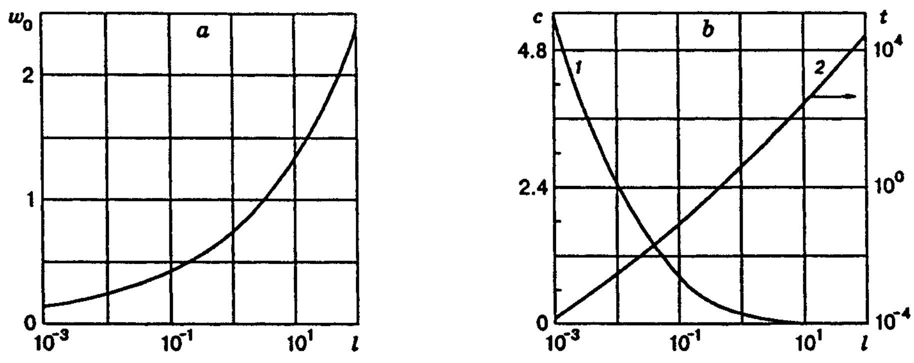

Fig. 1

the velocity c from the "matching" conditions of the numerical and asymptotic solutions. By construction, the latter is subject to the homogeneous boundary conditions specified at the tip.

Figure 1 shows calculation results concerning the basic parameters of a growing crack: the dependence of the crack opening in the center $w_0 = w(l, l)$, calculated for an injection regime (Fig. 1a), and the crack velocity c (curve 1) and the injection time t (curve 2) versus the length l (Fig. 1b).

3. We shall consider the crack behavior after the well is closed. A perturbation, which is generated by a change of the regime, propagates over the crack with a finite velocity. Therefore, the crack growth does not stop instantly and proceeds until the perturbation reaches the crack-tip vicinity. After that, the flow from the basic part of the crack decreases and fluid losses begin to play the major role. While the growth continues, the crack can be considered within the framework of the scheme described above. From a formal point of view, the only difference is the zero flow rate in the center [the last boundary condition (2.2) will become homogeneous]. However, when the crack reaches a maximum size, the asymptotics at the tip varies markedly. It requires an appropriate modification of the calculation scheme.

We shall study the behavior of the opening in this case. We make an assumption, which will be justified in our subsequent consideration, that the boundary layer at the tip disappears and $\partial w/\partial\xi$ is limited at the point $\xi = 0$. By virtue of the boundary condition $w(0) = 0$, in a small vicinity of the tip, the quantity w is of the order ξ, and the derivative $\partial w^4/\partial\xi$ is of the order ξ^3. Consequently, the local flow rate q tends to zero at $\xi \to 0$ as ξ^3. Thus, both homogeneous boundary conditions are automatically satisfied. For small ξ, the derivative $\partial q/\partial\xi = \partial w/\partial t + 1/\sqrt{t - t'}$ tends to zero as ξ^2, and, hence, the derivative $\partial w/\partial t$ at the boundary for $x = l$ is equal to $\partial w/\partial t = -1/\sqrt{t - t'}$. We emphasize that the moment t' refers to the injection regime, and $t - t'$ does not vanish because the derivative has no singularity.

Below, it will be convenient to regard w as a function of x and t. Let $x(t)$ be the law of motion of an edge. By definition, the opening at the edge is equal to zero at any moment of time and, therefore, the condition $dw = (\partial w/\partial x)\,dx + (\partial w/\partial t)\,dt = 0$ is satisfied at the edge, and the velocity of motion of the tip c is representable in the form

$$c \equiv \frac{dx}{dt} = 1\Big/\left(\sqrt{t - t'}\,\frac{\partial w}{\partial x}\right). \tag{3.1}$$

With decrease in the length, the crack opening decreases to zero at a point at which the tip arrives, and the derivative $\partial w/\partial x$ is negative at this moment. According to (3.1), the velocity c is negative as well, as it should be at the relaxation stage, i.e., at the stage considered the above assumption of the absence of a boundary layer at the top is true.

At the relaxation stage, it is convenient to formulate the problem in the form (1.6) and (1.7), together with the last condition (1.8) for $q_0 = 0$. Passing to a consideration of the current state, we assume the previous state of the crack, in particular, the gradient of opening at the edge, to be specified. Then, using formula (3.1), it is possible to find the velocity of the boundary at this moment, which will allow one to determine its current position as well. Thus, we come to a problem with known boundaries, one boundary condition being

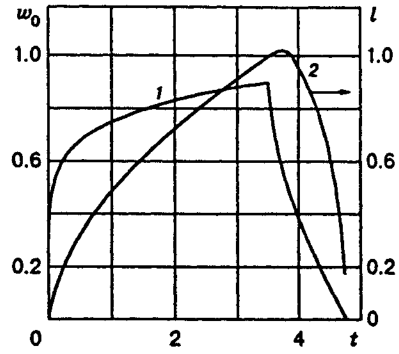

Fig. 2

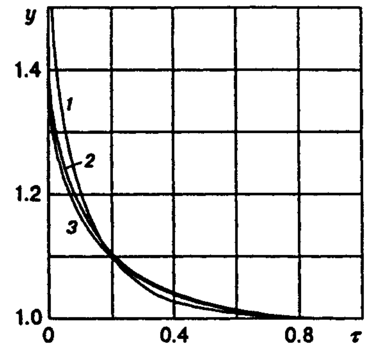

Fig. 3

given at each boundary. We emphasize that, in the case considered, the derivatives and the right-hand side of in Eq. (1.6) have no singularities, and this allows us to employ the sweep method throughout the domain without any difficulties. The boundary condition for the flow rate at the edge is not used, but this is connected with the fact that this condition is satisfied automatically in the absence of a boundary layer.

Figure 2 shows calculation results for the opening w_0 in the center (curve 1) and the length l (curve 2) as a function of time (at first, in an injection regime until the value $l = 1$ is reached and then for the zero flow rate until the crack is completely closed).

As we have already mentioned, the curve $w_0(t)$ is universal for injection. It is natural that the behavior of the opening for the zero flow rate depends on the injection time t_0. However, after the perturbation of the crack profile, which is caused by a change of the regime, smooths out in its central part and the velocity of a fluid flow to the periphery decreases considerably, the subsidence in the center again will gain a universal character. Let us consider the Nordgren equation in a small neighborhood of the center. The smoothness of the profile means that the derivative $\partial^2 w/\partial x^2$ becomes a small quantity, so that, for small x, the Nordgren equation takes the form

$$\frac{\partial w}{\partial t} = -\frac{1}{\sqrt{t-t'}}, \tag{3.2}$$

from which it follows that the rate of subsidence in the center vicinity is determined only by fluid losses.

We introduce the variable τ:

$$\tau = (t - t_0)/(t_c - t_0). \tag{3.3}$$

The moments t_c and t_0, which correspond to the full crack closure and the cessation of injection, respectively, are regarded here as the functions of the crack length l_0 at the moment t_0.

Assuming that $x = 0$ in (3.2) and, hence, $t' = 0$, integrating over t, and passing to the variable τ, we obtain $w_0 \approx 2\sqrt{t_c} - 2\sqrt{t_0 + (t_c - t_0)\tau}$. It follows that, for $\tau \to 1$, the function $y(\tau) = w_0(\tau)/\left(2\sqrt{t_c} - 2\sqrt{t_0 + (t_c - t_0)\tau}\right)$ tends to unity. As is seen from Fig. 3, in the coordinates y, τ the curves of opening reach almost simultaneously the common asymptotics in a broad range of l_0 values (curves 1, 2, and 3 refer to $l_0 = 0.01$, 0.1, and 1.1, respectively).

4. For a sufficiently intense outflow, there is a qualitative distinction in the crack behavior in different regimes, which is desirable to analyze. Of special interest is the effect of fluid losses on the quantity to be measured, i.e., the pressure in the center of the crack. To do this, we should make a preliminary analysis of injection and outflow in the limiting case of an impermeable medium.

The equation and the initial and boundary conditions in this case look like

$$\frac{\partial^2 w^4}{\partial x^2} = \frac{\partial w}{\partial t}; \tag{4.1}$$

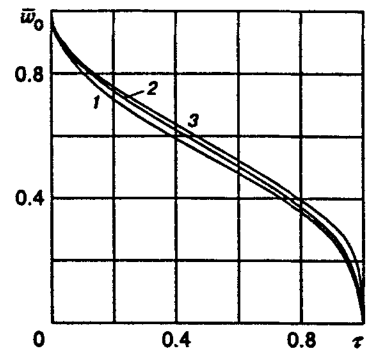

Fig. 4

$$w\Big|_{t=0} = 0, \qquad w\Big|_{x=l} = 0; \tag{4.2}$$

$$q(x,l)\Big|_{x=l} = \frac{\partial w^4}{\partial x}\Big|_{x=l} = 0; \tag{4.3}$$

$$q(x,l)\Big|_{x=0} = -\frac{\partial w^4}{\partial x}\Big|_{x=0} = 1 \quad \text{during injection,} \tag{4.4}$$

$$q(x,l)\Big|_{x=0} = -\frac{\partial w^4}{\partial x}\Big|_{x=0} = -\alpha \qquad (\alpha \sim 1) \quad \text{and during outflow.} \tag{4.5}$$

In an injection regime, the set of subsequent states of the crack can be ordered using any monotonically varying parameter, for example, the magnitude of opening in the center. The state at the moment of injection termination can be characterized by the parameter w_{00}, which is the value of half-opening in the center. Generally speaking, in an outflow regime the opening depends on w_{00}. In contrast to the general case, for an impermeable medium this dependence is found analytically. During injection and flowback, the solution for an arbitrary value of w_{00} is reduced to the solution for some fixed value, for example, for $w_{00} = 1$. This follows from the invariance of the entire system (4.1)–(4.5) relative to the scale transformations $w \to w_{00}w$, $x \to w_{00}^4 x$, and $t \to w_{00}^5 t$.

If one considers the solution for $w_{00} = 1$ and a usual [1] ratio of the flow rates during flowback and injection $\alpha = 0.25$, the injection duration is $t_0 = 0.9735$ and the flowback duration is $t_c - t_0 = 0.8122$, where $t_c = 1.7857$ is the moment of crack closure in the center, which is assumed to be the moment of flowback cessation.

For $w_{00} \neq 1$, we denote the ratio w_0/w_{00} by $\bar{w}_0$. It is convenient to pass to the variable τ defined by relation (3.3). Based on the above symmetry of Eqs. (4.1)–(4.5), we can state that in a flowback regime, the subsidence in the center is described, in the chosen coordinates $(\bar{w}_0, \tau)$, by the universal curve 1 of Fig. 4, which does not depend on the injection and flowback duration.

We shall express $\bar{w}_0$ and τ via the dimensional quantities $W(0,T) = W(X,T)\big|_{X=0}$ and T:

$$\bar{w}_0 = \frac{W(0,T)}{W(0,0)}, \qquad \tau = \frac{T - T_0}{T_c - T_0}. \tag{4.6}$$

The quantity $\bar{w}_0$ can be expressed also via the measured pressure. According to (1.1) and formulas (4.6), we have

$$\bar{w}_0 = \frac{P(T) - P_c}{P(T_0) - P_c}. \tag{4.7}$$

Together with the subsidence curve for an impermeable medium, Fig. 4 shows the dependence $\bar{w}_0(\tau)$ calculated within the framework of the general model (1.6)–(1.8) for the cases of small and moderate fluid losses ($l_0 = 0.1$ and 1 curves refer to 2 and 3, respectively). Clearly, all the curves are fairly close. This means that the fluid losses have no significant effect on the character of pressure drop $P(T)$ during flowback in layers of poor and moderate permeability.

At first sight, this insensitivity seems strange. Indeed, if the fluid losses are not negligible, they have an effect on the crack profile already in an injection regime, particularly after its completion (the formation of the initial crack profile for flowback) and necessarily affects the deformation of the profile during flowback. Nevertheless, the pressure drop in the center mainly depends on what happens in its close proximity. An intense flowback occurs sufficiently rapidly and for this time the perturbation induced by a change of the regime does not propagate far and, therefore, the crack does not stop even at the moment of its collapse in the central part. At the same time, the subsidence in the center is due to processes occurring only in this comparatively small perturbed zone. In the zone that we are interested in, the fluid losses are insignificant in comparison with the flow rate during flowback. Here, in forming the initial profile of the crack at the flowback stage, the rate of the total fluid losses is small as well in comparison with the flow rate during injection.

The universal specifics in the pressure drop during flowback are especially useful for calculation of the closure pressure P_c with the use of hydraulic fracturing data. Relation (4.7) includes the moment T_c of crack closure in the center and the pressure P_c, which is very difficult to find immediately from the measurements. However, if one considers the second equation in (4.6) together with (4.7) with the use of the $P(T)$ measurement data in a flowback regime, we obtain a system of algebraic equations that is sufficient for finding P_c at least, at two more points T_1 and T_2, in addition to the point T_0, with the known function $\bar{w}_0(\tau)$. It is possible to show that to improve the accuracy, the moment T_1 should be chosen before a break in the curve $P(T)$ occurs, but not too close to T_0, and the moment T_2 should be chosen after a break of the curve, but not too close to T_1 or T_c. It follows from the dependences in Fig. 4 that the pressure at the point of inflection of the curve $P(T)$ does not coincide with the closure pressure, which contradicts the popular opinion [1].

REFERENCES

1. M. J. Economides and K. G. Nolte (eds.) *Reservoir Stimulation*, Schlumberger Educational Serv., Houston (1989).
2. R. P. Nordgren "Propagation of a vertical hydraulic fracture," *SPE J.* **12**, 306–314 (1972).
3. L. F. Kemp, "Study of Nordgren's equation of hydraulic fracturing," *SPE Prod. Eng.* **5**. 311–314 (1990).